Chapter 6

Newton's Laws of Motion

6.1 Introduction

We have now considered units and dimensions, vectors, calculus, and a few simple motion problems generally described as kinematics. We are finally ready to discuss real physics.

Physics is the study of matter and motion, of forces and fields. Classical mechanics is based on the existence of particles, identifiable forces that act on those particles, and the response of the particles to those forces. The responses to those forces are described by Newton's three Laws of Motion. Newton worked in the 17th century, inventing the key mathematical tool, the calculus.

In the late 19th century, it became apparent that Newtonian mechanics did not appear to be entirely adequate. The specific heat of air, the specific heat of a vacuum within a metallic box, electrochemistry, the stability of atoms, the perpetual emission of heat by radium metal, the precession of Mercury's perihelion, and the tetrahedral bond arrangement of carbon in simple organic compounds all seemed beyond the ability of Newtonian mechanics to explain. In this century, quantum mechanics and general relativity explained these phenomena, which arise from processes taking place over very small distances, very short times, very high speeds, or in intense gravitational fields.

However, there are an extremely wide range of phenomena that are adequately described by Newtonian mechanics, because over macroscopic distances and speeds, not near stars, quantum mechanics and general relativity predict Newtonian behavior, at least very nearly.

Interpreting the Laws of Motion require some discussion of *reference frames*; we'll get that in the next chapter.

6.2 Newton's Laws

Newton's Laws of Motion begin with the existence of the quantity *momentum*. The momentum $\mathbf{p}$ is a vector. *In Cartesian coordinates*, the momentum of a particle is

$$\mathbf{p} = m\mathbf{v}. \tag{6.1}$$

In this equation, m is the mass of the particle and $\mathbf{v}$ is the particle's velocity. This equation satisfies the Curie principle. On the left-hand-side, $\mathbf{p}$ is a vector. On the right-hand-side, $\mathbf{v}$ is a vector, while $m\mathbf{v}$, being the product of a vector and a scalar, is also a vector. This equation equates two vectors, as is allowed by the Curie principle.

To repeat, equation 6.1 for the momentum *is only correct in Cartesian coordinates.* If you want to go to some other coordinate system, you have to use techniques beyond what is covered in this text to determine how to write $\mathbf{p}$.

Newton's three Laws of Motion are:

The Second Law, which tells us that

$$\frac{d\mathbf{p}}{dt} = \mathcal{F}. \tag{6.2}$$

in this equation, $\mathcal{F}$ is the total force on the mass m due to all forces acting on it. Forces are vectors and add as vectors.

The First Law (which is actually a corollary of the Second Law) tells us that if there is no force on an object, then the object continues to move in a straight line without changing its speed.

And, finally, there is **The Third Law**, which I will give as *all forces come in action-reaction pairs.* Newton gave a somewhat different phrasing for this law, one which is a bit hard to follow. You'll find that my phrasing and explanation of the Third Law makes things much clearer.

Why should we believe that Newton's Laws of Motion are correct, at least over a large range of times, distances, masses, and velocities? The answer is that they reduce an extremely large number of different natural phenomena to a coherent whole. They provide a very simple description for a large part of natural behavior. They are not complete. At large velocities, the behavior of two parallel lines of clocks moving with respect to each other is not quite what Newton had expected. These behaviors are correctly described by invoking special and general relativity. How large is large? Actually, one deviation from Newton's understanding of time has been measured using a very good clock and a jet airliner. By the time you have an object moving as fast as an earth satellite, the effects are significant for engineering purposes. The satellite-based Global Positioning System would not work correctly unless corrections due to special and general relativity were applied. Newton's Laws of Motion also do not work on small distance scales. If Newton's Laws of Motion were correct on extremely small distance scales, atoms and molecules as we know them could not exist. However, there is a wide range of phenomena, including most of engineering, in which deviations from Newton's Laws of Motion are of minimal practical consequence.

We begin with Newton's Second Law, which tells us that the time rate of change of the momentum is determined by the total force on the object. The components of that equation read

$$F_x = \frac{dp_x}{dt}, \tag{6.3}$$

$$F_y = \frac{dp_y}{dt}, \tag{6.4}$$

$$F_z = \frac{dp_z}{dt}. \tag{6.5}$$

I've use the usual notation for the Cartesian components of the vector $\mathcal{F}$. This second set of equations begs the question of how we know what the forces on an object are. We'll get to that later.

In many problems, the mass of the object being considered is a constant that does not change in time. That's not always true; for example, when a rocket takes off and flies into orbit, it burns fuel as it accelerates. It becomes lighter and lighter as it climbs. If the mass of the object being considered does not change as time passes, we may rewrite Newton's Second Law as

$$F_x = m\frac{dv_x}{dt} = m\frac{d^2x}{dt^2}, \tag{6.6}$$

$$F_y = m\frac{dv_y}{dt} = m\frac{d^2y}{dt^2}, \tag{6.7}$$

$$F_z = m\frac{dv_z}{dt} = m\frac{d^2z}{dt^2}. \tag{6.8}$$

Mathematically, the First Law is simply a corollary of the Second Law. If the total force on an object is zero, then equation 6.8 becomes

$$0 = m\frac{d^2x}{dt^2}, \tag{6.9}$$

$$0 = m\frac{d^2y}{dt^2}, \tag{6.10}$$

$$0 = m\frac{d^2z}{dt^2}. \tag{6.11}$$

The accelerations (the second time derivatives of the position) are all zero. Dividing out the m and integrating these three equations with respect to time, we obtain three constants of integration v_{0x}, v_0, and v_{0z}.

$$\frac{dx}{dt} = v_{0x}, \tag{6.12}$$
$$\frac{dy}{dt} = v_{0y}, \tag{6.13}$$
$$\frac{dz}{dt} = v_{0z}. \tag{6.14}$$

According to Newton's First Law, the velocity of an object subject to a net force of zero is a constant.

One might then ask why Newton called these two Laws the Second Law and the First Law, rather than numbering them the other way around. Part of the answer is that Newton did not work in a vacuum. Prior to Newton, there was an extensive intellectual belief as to the nature of motion. Newton rejected a large part of this belief.

Some authors will trace the theory of motion in question back to Aristotle, but Aristotle's idea of change was very different from anything we would recognize as a derivative with respect to time. In defense of Aristotle, he did not know calculus or algebraic geometry. Indeed, the Greeks did have excellent plane and solid geometry, but they were reasonably careful most of the time not to use diagrams in their proofs. Diagrams mislead, because they provide a specific example of a class of objects. It is then very easy to incorporate properties of that specific example into a proof, even though those properties are concomitant and not true for all objects in the class. Aristotle had one modest advantage over the ancient Romans, namely the Greek system for writing large numbers was much better than the Roman system, at least if you wanted to write a large number, say the number of grains of sand in a bucket.

[Aside: Why did the Greeks avoid diagrams? Consider a parallelogram with a line connecting the two opposite corners. The line divides the parallelogram into two similar triangles. That's a standard middle school plane geometry problem. However, that middle school proof actually contains an enormous hole. The hard part of the proof is to prove that the line between the two opposite corners lies someplace inside the parallelogram, so that it actually divides the parallelogram to two triangles, as opposed to lying outside the parallelogram, thus failing to divide the parallelogram into two triangles.]

What was the pre-Newtonian, pre-Galileo belief as to the nature of motion? A substantial body of thinkers believed in the notion of *impetus*. You fired a cannon at a wall. The cannon transferred to the cannonball a quantity of impetus. The cannon ball traveled through the air. As it moved, it exhausted its supply of impetus. When it ran out of impetus, it crashed to the ground. You may think of impetus as functioning like the fuel for a jet airliner. The airliner flies along until it runs out of fuel. It then goes into a glide until it hopefully reaches a runway. (At least one modern western airline has managed to do this. Twice.)

In Newton's time, there were good reasons to believe in the impetus model. You fired a cannon at a besieged fortress, while standing a safe distance behind the cannon. You saw the cannonball sail out, climbing as it went, and at some point the cannonball appeared to drop quite quickly to earth. Part of this sudden drop with less forward motion is an optical illusion. Part of this is that period cannon balls were very poorly made spheres, so that cannon balls had very large amounts of drag due to the air and did indeed slow down at the far end of their trajectories. To make matters more challenging, period muskets and cannon could not be aimed effectively. A standard European manual on siege cannon – you fire cannon balls at a wall until is has a hole in it – estimates that a cannon ball strikes the wall within something like fifteen degrees of the direction in which the cannon was pointed.

Galileo Galilei had done experiments which more or less demonstrated the First Law, the Law of Inertia, which says that an object moving under the influence of no forces will simply keep on moving. He experimented with rolling a smooth sphere down a ramp and then letting it climb back up a second ramp. He observed that if you made this second ramp shallower and shallower the sphere would still travel until it reached the same height as its starting point. (You may recognize this as the law of conservation of energy.) He reasoned that if the second ramp were made horizontal the rolling sphere would never get back to its original height and therefore would keep on going indefinitely. (Actually, there is something called rolling friction that will eventually bring the sphere to a stop.)

The other significance of the First Law is that it says there are coordinate systems in which if there is no force the velocity remains constant. When we discuss reference frames, we will come back to this point.

And now we reach the *Third Law.* My statement of the Third Law is quite different from the one you will have encountered elsewhere, the statement represented as an exact translation of Newton's words. The form you will read here does have exactly the same content. However, the point of having the Third Law is to get across a certain amount of information, not to repeat the not very good English translation, from centuries ago, of Newton's original book, which was, of course, written entirely in Latin, in the form of a plane geometry text.

The Third Law tells us:

All forces come in action-reaction pairs. If we call the two forces of an action-reaction pair $\mathbf{F}_1$ and $\mathbf{F}_2$, their properties are as follows.

1) The forces of an action-reaction pair are always on two different bodies. Two forces on the same body are never an action-reaction pair.

2) The forces in an action-reaction pair are equal in magnitude, so that $F_1 = F_2$. The forces in an action-reaction pair point in opposite directions, so that $\hat{\mathbf{F_1}} = -\hat{\mathbf{F_2}}$.

3) As a result, $\mathbf{F}_1 = -\mathbf{F}_2$.

4) The forces in an action-reaction pair have the same physical basis. What do I mean "physical basis"? The answer will become clearer after further discussion.

5) The action and reaction forces are simultaneous. By 'simultaneous' I mean that it is incorrect to ask which force is the action force and which force is the reaction force. The two forces always come as a pair. If you call one of the two forces the "action force", then the other of the two forces is the "reaction force", but it does not matter which of the two forces you called the action force. (Aside: Some of you have already heard about special relativity, and the question of what 'simultaneous' means in comparing clocks. I am here using 'simultaneous' in a completely different sense.)

As a modest qualification, if one studies intermolecular forces there are also *three-body forces*, in which three molecules put a force on each other that cannot be split into a trio of pair forces. Three-body forces are actually quite important in properties of liquids, but for this course we will stay with two-body forces.

Critics of Newton propose that Newton's laws are actually content-free. That is, the way we know that there is a force is that there is a response, an acceleration or other forces, and therefore the Second Law is a definition of 'force', not a law of nature. We could imagine how this claim could be true. People have put into orbit around the earth thousands of earth satellites. If the force between each earth satellite and our planet was different from the force on each of the other satellites, so that corresponding to each earth satellite there was a different law of gravity determining its orbit, the laws of gravity would be little more useful than a list of the positions and velocities of each orbit at different times. That's not what happens. Instead, there is one law of gravity, with all satellites being subject to the same law of gravity. There is a single gravitational force that rules them all and in their orbits binds them. By introducing the correct forces, we take a huge numerical description of planetary orbits, and reduce it to a few equations and a relatively small number of constants. That's a strong argument that forces are real, and that we more or less know what they are.

The reality, a third of a millennium after Newton did his work, is that a very small number of forces and a few other bits and pieces describe a huge number of natural phenomena. The traditional list of forces that I learned as an undergraduate, is

- The strong nuclear force.
- The weak nuclear force.
- The electromagnetic force, as explained by Maxwell and Dirac.
- Gravity, as explained by Newton and Einstein

Separate from these is the Pauli exclusion principle, which constrains how electrons can move, but is not a force.

Now an amusing historical anecdote arises. I was at a small student seminar run by the physicist who supervised my bachelor's thesis. He came in with this neat new experiment he had just invented, and told no one else about yet, a device for detecting gravity waves, a definitive test of Einstein's General Relativity theory of gravity. We were the first people to hear about it. Four decades, thousands-of man-years, and a hundred million dollars later, the Large Interferometric Gravitational Observatory worked as hoped, and Rainer Weiss shared in the Nobel Prize for inventing it and making it work.

6.3 Applications of Newton's Laws

We now turn to some simple applications of Newton's Laws. As a simple example, consider me, having mass m, standing on a chair. I'll first show the process, and then explain what the steps were.

What do Newton's Laws tell us? We start by drawing a sketch, and identifying the forces acting on the person. There are two of them, the force of gravity $-mg\hat{\mathbf{k}}$ pulling the person downward, and the *normal force* $\mathbf{N}$ due to the chain pushing him upward. Gravity is always with us. The normal force may be less familiar. The normal force is a contact force; it arises because objects are touching each other. It keeps objects from moving through each other. $\mathbf{N}$ arises from the electron clouds surrounding atoms and molecules. The clouds very much resist moving through each other, so material objects cannot readily interpenetrate. The normal force can be explained in detail in terms of the forces on the previous page, but we will not do that here. $\mathbf{N}$ is called the *normal force* because it points normal (perpendicular) to the surface of the two objects that are touching each other.

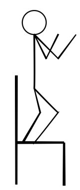

Figure 6.1: your author standing on a chair.

Let's consider the person standing on the chair. Like everyone else, the person is subject to a force of gravity $-mg\hat{\mathbf{k}}$. So long as the person stays on the surface of the earth, that gravitational force, the person's weight, stays approximately the same.

The normal force differs from many other forces in that it does not have a fixed value; the strength of the normal force is what you need to keep two objects from moving through each other. In the case at hand, the person is standing on the chair. The normal force points in the direction opposite to the force of gravity and is exactly strong enough that the person does not go through the chair. Quantitatively, the normal force is not stronger than the force of gravity, so the person is never pushed up away from the chair. The normal force is also never weaker than the force of gravity, so the person never floats ghost-like through the chair.

Let's consider these forces as parts of action-reaction pairs. There is a normal force of the chair pushing up on the person. That force has a reaction force, the normal force of the person pushing down on the chair. Those two forces satisfy Newton's Third Law. They are on two different objects, the person and the chair. They are equal in magnitude and opposite in sign; one pushes up, the other pushes down. They are of the same physical nature; they are both contact forces, the normal force.

There is also the gravitational force of the earth on the person. That force pulls the person down. What is the reaction force to the gravitational force of the earth on the person? Why, it's the gravitational force of the person on the earth. Those two forces are on two different objects, the person and the earth. They point in opposite directions, and as we will see later in the course are indeed equal in magnitude. They have the same physical nature; they are both gravitational. The force and reaction force act at the same time, so they are *simultaneous* in the sense of this Chapter.

You might reasonably question my claim that the two gravitational forces are equal in magnitude. After all, the planet Earth is much more massive than I am, so it has a much stronger gravitational field. However, the Earth's gravitational field can only act on my near-hundred kilograms. My gravitational field, while very weak, acts on the chair, the foundations of the building, the core of the earth, and the bright sunlit waves breaking across the windswept Indian Ocean. My gravitational field is acting on the huge mass of planet Earth, and therefore manages to create on the Earth a force equal in magnitude and opposite in direction to the force of the Earth's gravitational field on me.

Are the forces $\mathbf{N}$ and $mg\hat{\mathbf{k}}$ acting on the person an action-reaction pair? No! They are acting on the same body, and have different physical natures, so they cannot possibly be an action-reaction pair.

Minor aside: mg is the magnitude of the gravitational force on the person. g is the constant that converts m into the gravitational force. No matter that it is often done, it is incorrect to call g as used here *the acceleration of gravity* because, so long as the chair does not collapse, the person is not accelerating. He is subject to a force, but he isn't accelerating.

We now take this list of forces and use it to create a *force diagram* for the person. Force diagrams are not a part of Newton's Laws of Motion. You can perfectly well solve all classical mechanical problems without using them. Indeed, you can march all the way through the full series of physics degrees and never hear of them. After all, I did. However, the force diagram is a useful mnemonic tool. It helps you confirm that you have identified all the forces in the problem and which bodies they are acting on.

In a force diagram, you represent the mass of interest by a large dot labeled with the mass of the object of interest. You now indicate on the force diagram all of the forces acting *on* the object of interest, representing each of them as an arrow pointing from the object in some direction, and a label indicating the magnitude of the force. In some cases, you actually don't know which way the force is pointing, so the arrow is inserted without believing that the force necessarily points in that direction. It might be the case that several forces are acting on a mass, all pointing in the same direction. The appropriate response is to show them deviated slightly to one side or the other so that each force is clearly visible. Figure 6.2b gives an example. The general rule is that a force diagram is a qualitative sketch of the problem, not a scale drawing. Finally, you indicate on the force diagram the coordinate axes being used to solve the problem.

Here are force diagrams for the person of interest and for a toy helium balloon.

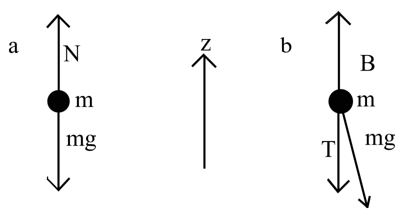

Figure 6.2: Two force diagrams. (a) Force diagram of the man standing on a chair. (b) Force diagram of a balloon on a string, including the buoyant force **B** due to the helium in the balloon, the force of gravity $-mg\hat{\mathbf{k}}$ on the balloon and the tension $-T\hat{\mathbf{k}}$ exerted on the balloon by its string. The two downward forces are exactly parallel to each other, but they are drawn slightly splayed with respect to each other so that they are both clearly visible.

The arrow pointing up labeled with the letter z is the vertical, z, coordinate. The black dot labeled m is the person. The two arrows pointing up and down are labeled N and mg. The labels could be the force components, or the magnitude of the forces. Writing the force of gravity on the force diagram as $-mg$ instead of $+mg$ is of no consequence. When we insert a force into a Second Law equation, the sign given to each term is determined by comparison between the direction of the force and the direction of the coordinate axes.

The force diagram emphatically *does not* include an arrow labeled $m\mathbf{a}$ to represent the mythical 'force of acceleration' or 'force of inertia'. There are no such forces, and therefore they are not shown on the force diagram. The choice of coordinate directions is up to the person solving the problem. Some choices of coordinate directions make a problem easier to solve. Some choices of coordinate directions make a problem more challenging to solve. However, a problem that is soluble with one choice of coordinate directions is equally soluble (the solution may look a bit different) with any other choice of coordinate directions.

We now invoke the Second Law and do substitution of the known forces. Starting with the Second Law, we have

$$m\frac{d^2\mathbf{r}}{dt^2} = \mathcal{F} \tag{6.15}$$

from which the z-component is

$$m\frac{d^2 z}{dt^2} = F_z. \tag{6.16}$$

The force F_z can be read off the force diagram. One inserts a list of all forces pointing along the z-axis, and the z-component of any force pointing in some other direction. Each force must be inserted with the correct sign so that it is pointing in the right direction. In the case at hand, one obtains

$$m\frac{d^2 z}{dt^2} = N - mg. \tag{6.17}$$

The person is not accelerating up or down, so d^2z/dt^2 is zero. Inserting this result into the Second Law, we obtain

$$N - mg = m \cdot 0, \tag{6.18}$$

leading to the solution $N = mg$. The normal and gravitational forces on the man are of equal magnitude and point in opposite directions, but they are not an action-reaction pair, because they act on the same object and have different physical natures.

A simple lecture demonstration brings this out. For the case of the hypothetical lecture-demonstration, I am standing on the chair. The legs of the chair have been carefully wired with small amounts of plastic explosive. The plastic explosive is now detonated. The legs are reduced to tiny particles that recede in all directions. Of course, if this were physics as seen in children's cartoons, I would be standing on the chair, perfectly safely, until I looked down and noticed that the legs of the chair were no longer present. I would then fall. We are not doing cartoon physics, we are doing real physics. The moment we detonate the legs of the chair, the chair can provide no support, so I would begin to fall groundward.

Figure 6.3 shows the explosion and the force diagram. Note the direction that $+z$ is assigned in the figure, namely **straight down**. There is no physics reason for choosing this direction. The reason I reoriented the coordinate axis is to demonstrate that the signs chosen for different forces are not intrinsic; they are determined by the directions of the coordinate axes. In this case, gravity is pointing in the $+z$ direction, as is my acceleration, while the normal force is $-N\hat{\mathbf{k}}$, so the z-component of the Second Law becomes

$$-N + mg = mg. \tag{6.19}$$

This equation has as its solution $N = 0$. N and mg are thus shown to be independent.

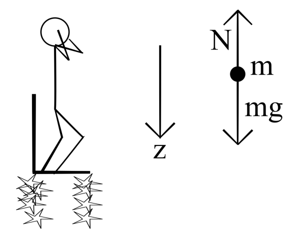

Figure 6.3: The chair legs have exploded! I fall downward at an acceleration g. A force diagram for this circumstance is displayed.

We now turn to a second problem illustrating the Laws of Motion. It's a propeller airplane pulling a glider. What is the acceleration of the glider, given that there is a force $\mathbf{P}$ on the airplane centered at the propeller? I begin by drawing a sketch of the problem, showing the airplace and glider, and collecting in one place all the information that I appear to need. The figure shows the propeller airplane, labeled '1' with a propeller in front and a tow rope tying it to a glider. The glider is labeled '2'. The masses of the airplane and glider are m_1 and m_2, respectively.

I now set up force diagrams for the airplane and the glider. In the vertical direction, the airplane and the glider are subject to gravitational forces $m_1 g$ and $m_2 g$ pointing down and lift forces L_1 and L_2 pointing up. In addition, the glider is subject to a force due to the tension T in the tow rope, that force pulling the glider in the $+\hat{\mathbf{i}}$ direction. The tension T also acts on the airplane in front, pulling on it in the $-\hat{\mathbf{i}}$ direction.

Finally, there is a force $\mathbf{P}$ present because the aircraft's propeller is spinning rapidly. The force that acts on the airplane is a force due to the air pushing on the propeller blade. The reaction force to that force on the airplane is a force due to the propeller blade pushing on the air. The force that moves the aircraft is the force on the propeller, this being the force of the air on the propeller. The force that the propeller puts on the air does move the air, but that is not a force on the airplane, so it does not contribute to moving the airplane.

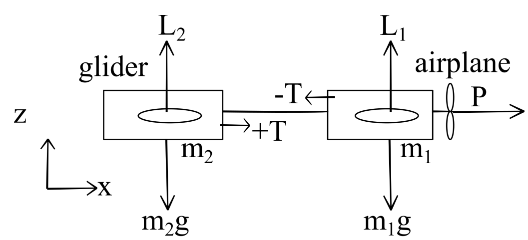

Figure 6.4: This figure is a *sketch* for the glider problem. *It is not a force diagram!* I indicate the airplane with propeller, headed right, the glider being towed behind, and the forces on each of them. The rope exerts a tension force on both vehicles. The sign on T is only a reminder to whoever is solving the problem that the two tension forces do not point in the same direction.

Let us pause to say something about tension. Tension is the force created by a solid object, such as a string or a rope, when it is stretched. On a microscopic level, when the object is stretched, the component atoms and molecule change their shapes, orientations, and distances apart, so that the object tries to pull its two ends in toward the center. If the object is very light, the forces it exerts at its two ends, on whatever is holding it, are very nearly equal in magnitude and opposite in direction. The forces exerted by a rope are always parallel to the rope. In the case here, where the rope is very light relative to the aircraft and the glider, the tension forces on the two ends of the rope are equal in magnitude. We will discuss 'very light' in a future chapter. The tension force never pushes away from the center of the rope, the principle being 'You can't push on a rope.'

We may now draw the force diagrams for the glider and the airplane. In the force diagram, each mass is indicated as a point. It is incorrect, because it leads to deceptive and wrong logic, to replace the point with a little sketch of the airplane. We need one force diagram for the glider and a separate force diagram for the airplane. It is simplest, but not required, that we put the coordinate axes in the same directions for the glider and for the airplane. We could switch one of them around, for example making $+z$ to be in the downward rather than the upward direction for the glider, but this leads to confusion without in this case netting any advantage.

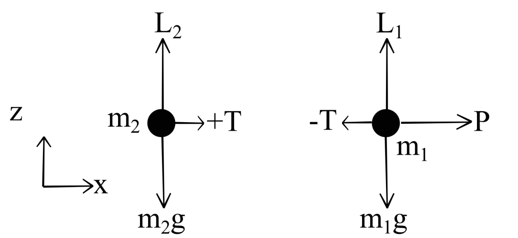

Figure 6.5: The force diagram for the glider problem,

We now write the Second Law for the two aircraft. To do things slightly differently, I will write the Second Law in its vector form. There is no new physics here relative to writing the Second Law in its scalar, component-by-component, form. In some problems, writing the Second Law componentwise makes life easier. In some cases, you are much better off writing the Second Law in its vector form.

From the force diagrams we obtain two force equations, one each for the propeller plane and the glider.

$$-T\hat{\mathbf{i}} + \mathbf{P} + L_1\hat{\mathbf{k}} - m_1 g\hat{\mathbf{k}} = m_1 \frac{d^2\mathbf{r}_1}{dt^2}, \tag{6.20}$$

$$+T\hat{\mathbf{i}} + L_2\hat{\mathbf{k}} - m_2 g\hat{\mathbf{k}} = m_2 \frac{d^2\mathbf{r}_2}{dt^2}. \tag{6.21}$$

In writing these equations, there is no reason to suppose that the lift forces on the two aircraft, the masses of the two aircraft, or the position vectors of the two aircraft are the same. To distinguish between them, I have labeled L, m, and r with subscripts, the numbers 1 and 2 referring to the propeller plane and the glider, respectively.

How do we solve these equations? Suppose we specify that the aircraft and glider are in level flight, with a known $\mathbf{P}$, pointing horizontally, that causes the aircraft and glider to accelerate in the horizontal direction. The masses m_1 and m_2 of the airplane and glider are take to be known. In level flight, the vertical components of the two accelerations are zero, so the vertical components of the two Second Law equations are

$$L_1 - m_1 g = m_1 \cdot 0, \tag{6.22}$$

$$L_2 - m_2 g = m_2 \cdot 0. \tag{6.23}$$

Here the two vertical accelerations were replaced by their values, these being zero. We have two equations and two unknowns, so we expect to be able to solve for L_1 and L_2.

The horizontal components of these equations are

$$-T + P = m_1 \frac{d^2 x_1}{dt^2}, \tag{6.24}$$

$$T = m_2 \frac{d^2 x_2}{dt^2}. \tag{6.25}$$

If we look carefully, we see there is a problem. P, m_1, and m_2 are all known. However, that leaves us with three unknowns, namely T and the two accelerations. We have therefore two equations and three unknowns. How can we solve?

The answer is that there is another equation, a *constraint*. Constraints put limits on the values of different variables. In the case at hand the constraint is that the distance between the airplane and the glider is fixed at some distance k that is determined by the length of the rope. This gives us a new equation

$$x_2 - x_1 = k. \tag{6.26}$$

Taking the second time derivative of this equation, we find

$$\frac{d^2 x_2}{dt^2} - \frac{d^2 x_1}{dt^2} = 0. \tag{6.27}$$

Equation 6.27 is a third equation connecting the three unknowns, so we can solve for T and the accelerations. The constraint that the accelerations of the airplane and glider are equal, rather than having, e.g., different values in a fixed ratio, is an accident: This constraint happens to be true in this particular problem. In another problem, there may be a different constraint. Finally, note that we omitted any drag force on the airplane and glider; that neglect is not a good approximation here.

6.4 Discussion

We have now introduced Newton's Three Laws, which tell us

First, the Second Law

$$\mathcal{F} = \frac{d\mathbf{p}}{dt}. \tag{6.28}$$

Then, the First Law, which is simply a corollary of the second: If $\mathcal{F} = \mathbf{0}$, then the momentum $\mathbf{p}$ is a constant.

Finally, the Third Law: All forces come in Action-Reaction pairs. If we call the two forces of an action-reaction pair $\mathbf{F}_1$ and $\mathbf{F}_2$, their properties are as follows.

1) The forces of an action-reaction pair are always on two different bodies. Two forces on the same body are never an action-reaction pair.

2) The forces in an action-reaction pair are equal in magnitude, so that $F_1 = F_2$.

3) The forces in an action-reaction pair point in opposite directions, so that $\hat{\mathbf{F_1}} = -\hat{\mathbf{F_2}}$. As a result, $\mathbf{F}_1 = -\mathbf{F}_2$.

4) The forces in an action-reaction pair have the same physical basis. What do I mean "physical basis? The answer will become clear after further discussion.

5) The action and reaction forces are simultaneous.

Having said this, there is a simple grammatical trick for identifying the reaction force to a specified force. The specification of a force indicates the symbol for the force, what type of force we are talking about, the source of the force, and the target of the force. Thus, mg is the magnitude of the force of gravity of *the earth* on **the man**. The reaction force to the force of gravity on the man is a force of magnitude mg of **the man** on *the earth.*

To find the reaction force, all we have to do is to exchange the source of the force and the target of the force. Because the action and reaction forces are simultaneous, we could then write: The reaction force to the force of gravity mg by the man on the earth is a force of magnitude mg of *the earth* on **the man**.

6.5 Worked Problems

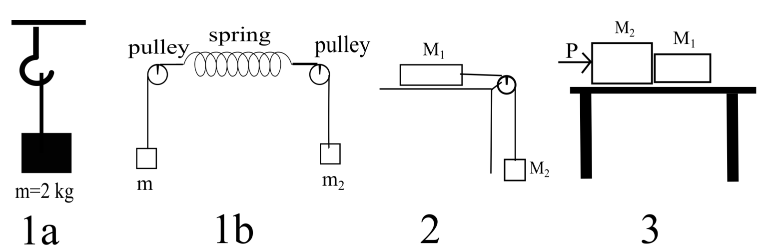

Figure 6.6: Figures for Worked Problems 1, 2, and 3.

1. (a) Figure 6.6, part 1a, shows a 2 kg mass hanging at the end of a rope. The top of the rope is wrapped securely around a hook that is attached to the ceiling. For the 2 kg mass, draw the force diagram. *In complete sentences*, identify each of the forces acting on the mass, including the nature of the force and the object applying it. In complete sentences, for each force in your force diagram, identify the reaction force, the object applying it, the nature of the force, and the object on which the reaction force is acting. (b) Figure 6.6, part 1b, shows two masses suspended by ropes; the ropes at their tops ends each go over a pulley and are connected by a spring. Repeat part (a) of this problem for the left-hand-mass and the left hand rope in the Figure. Hint: Including its own weight, there are four forces acting on the rope. The answer to this problem is quite long but should not be complicated. Hint: The phrase "complete sentence" has an exact meaning.

2. For the masses shown in Figure 6.6, part 2, find the force diagrams for the two masses, and for each force identify the object applying it, the nature of the force, and the object on which the force is acting.

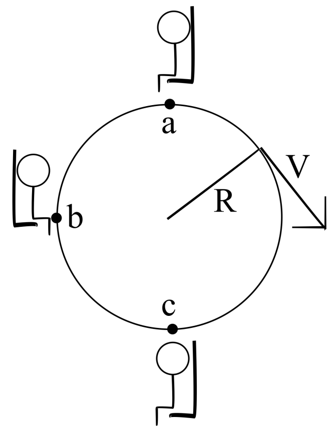

Figure 6.7: The Ferris Wheel problem.

M_1 and M_2 are connected by a rope that goes over a pulley, the pulley being represented in the figure by a circle.

3. For the masses shown in Figure 6.6, part 3, find the force diagrams for masses M_1 and M_2, and for each force identify the object applying it, the nature of the force, and the object on which the force is acting. The masses are resting on a horizontal table surface. [Hint: The force **P** pushing M_1 to the right is "external"; its nature and source cannot be determined from the problem as given.]

4. A stereo speaker is suspended from the ceiling by two wires. The speaker has mass m; the tensions in the two wires are T_1 and T_2. Draw the force diagram for the speaker. In complete sentences, identify each of the forces acting on the speaker. In complete sentences, for each force in your force diagram, identify the reaction force, the object applying it, the nature of the force, and the object on which the reaction force is acting.

5. An interesting example of a circular motion problem that turns out to be best solved by keeping the Second Law in its vector form is provided by that amusement park devisement, the Ferris Wheel. The wheel, radius R, is shown in Figure 6.7. Someone is sitting in a seat as the wheel goes around in a circle at constant speed v. What is the force on the person due to the chair at points a, b, and c? Hints: Two forces are acting on the person, a force **N** due to the chair, whose direction you do not know, and a force $-mg\hat{\mathbf{k}}$ due to gravity.

6.6 Homework

The first five problems all refer to Figure 6.4.

1. For the two aircraft in Figure xx, break the vector form of the Second Law into its components.

2. Solve for the tension in the rope and the acceleration of each aircraft.

3. The forces on the two ends of the rope due to the airplane and the glider are equal in magnitude, opposite in direction, and are both contact forces. Are they an action-reaction pair? Why not?

4. The lift force L_1 is the aerodynamic force of the air on the airplane. What is the reaction force to L_1?

5. For each of the forces acting on the airplane and the glider, identify the reaction force.

6. Calculate the acceleration of a falling body having mass m, using the skew coordinates indicated in Figure 6.8, in which the x and y axes are each at angle θ with respect to the horizontal or vertical, respectively.

7. A bow is used to fire an arrow into an arc in the air. Point a is after the archer has released the arrow, but before the arrow has left the bow. Point b is after the arrow has left the bow. Point c is the highest point reached by the arrow. Points d and e are on the way down. Write the free body diagram for the arrow at each point. What is the acceleration of the arrow at point c?

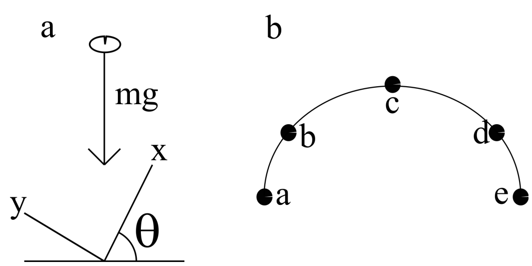

Figure 6.8: Figures for problems 6 and 7.

8. A man and a boy, masses M and m, respectively, both wearing ice skates, stand facing each other on wet, slick ice. The man pushes on the boy with a force $\mathbf{P}$ of magnitude 300 Newtons. Draw the complete force diagrams for the man and the boy. In complete sentences, specify the **reaction** forces corresponding to each force in your two force diagrams. Is the magnitude of the force with which the boy pushes back on the man smaller, larger, or the same as the force with which the man is pushing on the boy? In specifying a force, give the physical nature of the force, the object on which it is acting, and the object supplying the force.

9. A decorative sign is attached to the ceiling by two wires. The wires are tied to the sign at two different points, and slant away from each other and up toward the ceiling. Draw the complete force diagram for the sign. In complete sentences, specify the forces shown on your force diagram, and identify for each force its corresponding reaction force. In specifying a force, give the physical nature of the force, the object on which it is acting, and the object supplying the force.

10. (a) A bug collides with an automobile windshield. The bug exerts a force $\mathbf{F}$ on the windshield. The windshield exerts a force $\mathbf{F}'$ on the bug. What is the quantitative relationship between $\mathbf{F}$ and $\mathbf{F}'$? (b) The Earth exerts a gravitational force $-mg\hat{\mathbf{k}}$ on the car. Correspondingly, the car's wheels exert a force $N\hat{\mathbf{k}}$ on the earth. Are $-mg\hat{\mathbf{k}}$ and $N\hat{\mathbf{k}}$ an action-reaction pair? Why or why not?

11. A mass m hangs from a string. The far end of the string is attached to the ceiling. For the mass, and for the string, write the force diagram, labeling each force. Identify the reaction force to each force. In identifying each force and reaction force, specify the nature of the force, the object exerting the force, and the object on which the force is being exerted. Bonus solution: We have a mass m resting on top of another mass m', which in turn rests on the ground. Find the force diagrams for m and m', and specify the reaction forces to each of the forces in your diagrams.

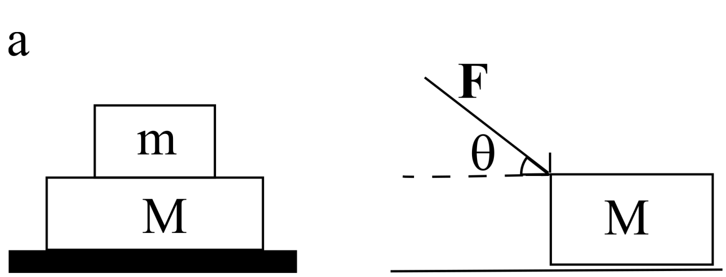

Figure 6.9: Figures for problems (a) 12 and (b) 13.

12. See Figure 6.9. A book having mass m is resting on a table having mass M in my office. The table is resting on the floor, which is part of the planet Earth. Find the force diagrams for the book, the table, and the Earth, and identify each force in a few words. For each force, identify the reaction force, including the nature of the force, the object exerting the force, and the target of the force. *Using the Second Law*, calculate the normal force exerted by the floor on the table.

13. See Figure 6.9. I push a trunk, mass M, along a well-waxed (frictionless) surface by applying to it a constant force $\mathbf{F}$ as shown. (a) Give the force diagram of the trunk. (b) Compute the normal force of the floor on the trunk. (c) Are (1) the force of the Earth's gravity on the trunk, and (2) the normal force of the floor on the trunk, an action-reaction pair? Why? (d) Obtain $x(t)$ and $v(t)$ of the trunk as functions of time, giving the most general correct forms for the solutions. (You may need to use some unknown constants.)

14. The new and improved rocket car has been given a new engine. The force that the engine applies on the car, over the time of interest, increases with time as $\mathbf{F}(t) = F_0 t^2 \hat{i}$. Here F_0 is a constant. The rocket car with engine has mass m. The rocket car is parked on flat horizontal ground. Find the force diagram. Find the acceleration of the rocket car, the velocity of the rocket car, and the position of the rocket car as functions of time.

15. The perpetrator of a prank gone wrong is hanging onto a rope a small distance out from the side of a building. The rope exerts on him a tension force of 800N straight up. He has a mass of 90 kg. Find his acceleration.

16. For the next scene in the movie, the lead actress is strapped into the rear seat of a high-performance combat aircraft, for whose use a considerable sum of money has been paid by the studio. [Some years ago, in a certain foreign country, you could actually do this.] The aircraft takes off and flies in a vertical circle at a constant speed of 300 m/s. The circle has a radius of 2000 m. At the top and bottom of the circle, and half-way in between, find the force that the aircraft exerts on the actress, who is taken to have a mass of 65 kg.

17. An automobile accelerates in the $+x$ direction in a straight line from a standing start, Draw a complete force diagram for the automobile. For each force on the automobile, identify the source and direction of the force.

18. We have a jogger on a running track. Discuss whether or not the following pairs of forces are an action-reaction pair: (a) The jogger's feet push down on the running track. The surface of the running track pushes up on the jogger's feet. (b) The earth's gravity pulls down on the jogger. The surface of the running track pushes up on the jogger's feet. (c) The jogger's feet push down on the running track. The earth's gravity pulls down on the jogger. (d) (b) The earth's gravity pulls down on the jogger. The jogger's feet push down on the running track.

19. Consider a crate resting on an inclined board. The crate is being pulled up the board by a rope that lies parallel to the board. Draw a complete force diagram for the crate. For each force on the crate, identify the nature of the force, the object exerting the force, and the direction of that force. Identify the reaction forces to the force on the crate. Identify the reaction forces to the forces on the crate, including the object exerting the force, the object on which the force is exerted, the physical nature of the reaction force, and the direction of the reaction force.

20. An experimental aircraft of mass M has a novel engine whose thrust depends on time as $T = T_o(1 - \exp(-\Gamma t))$, where T_o and Γ are constants determined by the details of the engine design. Here t is the time measured from the instant that the engine is started. The airplane is placed on a runway aligned along the x-axis and pointed in the $+\hat{\mathbf{i}}$ direction. The engine is started. This is a taxi test; the airplane does not leave the ground. The airplane may or may not have been at a stop when the engine is started. a) Find the x-component of the velocity of the aircraft as a position of time. b) Find the x-component of the position of the aircraft as a position of time.

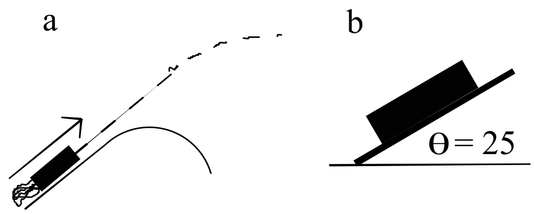

Figure 6.10: Figures for problems (a) 21 and (b) 22.

21. See Figure 6.10. Our rocket-propelled car, as described in lecture, reaches Worcester. In Worcester, it is to go up one of our steeper hills and over the top, where it goes airborne. (Sketch gives trajectory.) At $t = 0$, the car reaches the base of the hill, located at $(x, y, z) = (0, 0, 0)$. The car rolls up the hill at a constant speed (measured parallel to the hill) of 70 m/s. The slope of the hill is 45 degrees. At $t = 3$, the car reaches the top of the hill, located at $x = 150, z = 150$. At this point, the car leaves the ground, and the engine is turned off. The car moves under the influence of gravity until it hits the ground. For credit, you must use my coordinate origins to solve the problem. (a) Give the correct force diagrams for the car before and after the car leaves the ground. Identify each force with a short phrase. (b) For each force in your free body diagram, identify fully the corresponding reaction force. (c) A photographer wishes to take a picture of the car when it is 100m above the ground, and descending. The photographer needs to know the x coordinate and the exact time at which the car will be 100m above the ground. What x and t will give the photograph showing the car at this altitude?

22. See Figure 6.10b. A 200 kg desk is firmly bolted to a horizontal office floor. (i) Give the free body diagram for the desk. Label all forces, and identify each force in a short phrase. (ii) This is California. There is now an earthquake. The earthquake causes the office and desk to have a time dependent displacement which for most of the earthquake may be written

$$\mathbf{r} = 0.2\cos(7t)\hat{\boldsymbol{\imath}} + 0.1\sin(7t)\hat{\boldsymbol{k}} - 0.1(1 - \exp(0.8t))\hat{\boldsymbol{\jmath}}. \tag{6.29}$$

Here the x and z motion describe the earthquake oscillations, while the y motion refers to the local continental plate sliding toward the ocean. Find the velocity and acceleration of the desk during the earthquake. (iii) What was the total force on the desk during the earthquake? (iv) After the earthquake, the floor has gained a tilt of 25 degrees as shown in the sketch. Give the free body diagram for the desk. Label and identify in **complete English sentences** each of the forces on the desk. What is the reaction force to the force of the earth's gravity on the desk?

23. We now transport ourselves forwards to the twenty-second century, onto a manned space station in a low orbit around the earth. The station is a cubical box whose base always faces the earth. An astronaut is floating, stationary with respect to the walls of the space station, in the middle of the space station, not in contact with any object. Draw the free body diagram for the astronaut. Is the total force on the astronaut on the astronaut zero, or not? Why? Is the astronaut accelerating or not? Why?

24. A 4-kilogram mass is subject to a force $3\hat{\boldsymbol{\imath}}+7\hat{\boldsymbol{k}}t$ and a force $5\cos(\omega t)$ in the $+y$ direction. The frequency ω is 5 radians per second. At $t = 0$, the mass is stationary and located at the origin. (i) Find the position $\mathbf{r}$ and the velocity $\mathbf{v}$ of the mass as functions of time. (ii) What is the location of the mass at time $t = 2$?

25. It is a typical winter day in Worcester. The streets are all covered with two inches of polished ice, reducing friction to zero. The rocket-engine-powered car our second lecture has reached Worcester, and is trying to climb the hill on Salisbury Street located just west of Park Avenue. (i) Give the force diagram for the car. (ii) Write component by component the equations of motion $\vec{F} = m\frac{d^2\mathbf{r}}{dt^2}$ for the car. (iii) Find the minimum required thrust $\mid \mathbf{T} \mid$ (force applied to the car due to the rocket engine) if the car is to climb the hill at constant speed. (iv) If the car is to climb the hill at a constant road speed of 5 m/s, assuming a 5.0×10^3 kg car and a hill slope of 10 degrees, find as exactly as possible

the thrust of the rocket engine. (v) Find the normal force $\mathbf{N}'$ applied to the road by the car's tires. Assume that the tires carry the full weight of the car.

26. Consider an car accelerating up a steep hill. The wheels and engine are part of the car. (i) Give the force diagram for the driver. For each force, identify the nature of the force, the direction of the force, and the reaction force. I'll give you the first entry: Force mg is the force of the earth's gravity on the driver, pointing straight down. The reaction force is the force of the driver's gravity pulling up on the earth. (ii) Repeat part (i) of this problem, but this time give the force diagram for the car. For each force, identify the nature of the force, the direction of the force, and the reaction force. (iii) The car is now parked in a flat driveway. The forces on the car are the earth's gravitational force $-mg$, pointing straight down, and the normal force N due to the driveway, pointing up. Is N the reaction force to $-mg$? Why or why not? Explain in words, in one or more complete sentences.

27. This question asks for thought and discussion, not equations. Answer in complete sentences. Do not resort to writing equations in place of words. (i) According to Newton's Second Law, the vector sum of all forces acting on a body is equal to the mass of the body times its acceleration. What can we conclude about a body's motion if the vector sum of all forces acting on the body is zero? (ii) Some sources state Newton's third law in the following way: "When two bodies exert mutual forces on one another, the two forces are always equal in magnitude and opposite in direction." If the two forces are always equal in magnitude and opposite in direction, then their total is always zero. Consider your answer to part (i). Assume that Newton's laws of motion are correct. If the total force is always zero and $\mathbf{F} = m\mathbf{a}$, how is it possible for any object to be accelerated by forces due to other bodies?

28. A 37 kg mass hangs from the end of a spring whose equilibrium length is 2.00 m. The mass oscillates up and down over a distance $A = 10$ cm, at a frequency $\omega = 3$ radians/s, so that we may write the position of the mass as

$$z = A\cos(\omega t). \tag{6.30}$$

z is the displacement of the mass from its equilibrium position. What is the force F on the mass? (Hint: F depends on time.)

29. For each of the following, identify the correct answer, and explain why each choice is right or wrong: (i) An airplane ($m = 100,000$ kg, $v = 340$ m/s) collides with a moth ($m = 0.001$ kg, $v = 1$ m/s). During the collision, the airplane applies a force of magnitude F to the moth; the moth applies a force of magnitude f to the airplane. We may say with certainty that:

(i) $F > f$

(ii) $F = f$

(iii) $F < f$

(iv) Either none of the above are true, or more than one of the above could be true.

(ii) The same airplane ($m = 100,000$ kg, $v = 340$ m/s) now suffers a second and more serious collision, this with a giant flying creature escaped from a 1950's sci-fi thriller. The giant flying creature is large and fast (wingspan = 300 m, $m = 30,000,000$ kg, $v = 2000$ m/s) During the collision, a few pieces of the airplane are knocked free and thrown violently upwards. For a piece having mass m of 1 kg, we may say with certainty that

(i) The force of gravity on the piece is substantially changed during the collision by the acceleration of the collision.

(ii) The force of the planet earth's gravity on the piece is larger in magnitude than the force of the piece's gravitational field on the planet earth.

(iii) During the collision, the force of gravity on the piece is approximately 10 Newtons straight down, so from $F = ma$ the piece accelerates at 10 m/s^2 straight down.

(iv) The above three statements are all false.

(iii) On a fine winter day, an automobile skids across the library parking lot. The parking lot is covered with sheet ice, so the brakes have no effect. Until the collision, the car is subject to no horizontal force.

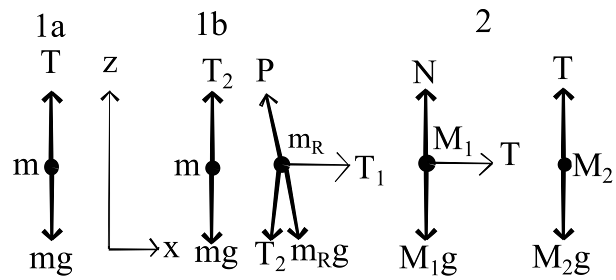

Figure 6.11: Force Diagrams for Worked Problems 1 and 2; labels 1a, 1b, and 2 correspond to those problems and parts..

The automobile then collides with a student on a skateboard. If the magnitude of the force the car exerts on the skater is F, and the magnitude of the force that the skater exerts on the car is f, we may say with certainty that

(i) $F > f$.

(ii) $F < f$.

(iii) Since there is no horizontal friction, the car and the skateboarder have no way to exert a force on each other.

(iv) None of the above statements is true, or more than one of the above statements could be true.

30. (i) In complete sentences or valid equations, state Newton's three laws of motion. (ii) List the properties that all action-reaction force pairs must have.

31. An A. D. 1940 jet aircraft, mass M, is parked at one end of the runway, preparing to take off. At time $t = 0$ the engine is engaged, and the aircraft begins to move down the runway. This is a very early jet airplane, so the engine takes from time $t = 0$ to some much later time T to come to full power. Between times 0 and T the thrust (the force on the airplane due to the engines) may be written

$$\mathbf{F} = \hat{\mathbf{i}}ct. \tag{6.31}$$

c is a constant. $\mathbf{F}$ depends on time. The aircraft remains on the ground until after time T. (i) Beginning with Newton's Laws of Motion, obtain explicit equations for the acceleration, velocity, and position of the aircraft as a function of time. Note that $|\mathbf{F}|$ is increasing with increasing time. (ii) At $t = T/2$, the aircraft's horizontal speed is 30 m/s. What is the aircraft's horizontal speed at $t = T$? (iii) At time $T/2$, the aircraft has travelled a distance $\Delta x = 300$m down the runway. How far down the runway has the aircraft travelled by time T?

6.7 Solutions to the Worked Problems

1. (a) $\mathbf{T}$ is the tension in the rope, a contact force applied to the mass. The reaction force to $\mathbf{T}$ is a contact force $-\mathbf{T}$ from the mass onto the rope. $-mg\hat{\mathbf{k}}$ is the force of the earth's gravity on the mass. The reaction force to $-mg\hat{\mathbf{k}}$ is the force $mg\hat{\mathbf{k}}$ of the mass's gravity on the earth.

   (b) Mass m: $-mg\hat{\mathbf{k}}$ is the force of the earth's gravity on the mass. The reaction force to $-mg\hat{\mathbf{k}}$ is the force $mg\hat{\mathbf{k}}$ of the mass's gravity on the earth. $\mathbf{T}$ is the tension in the rope, a contact force applied to the mass. The reaction force to $\mathbf{T}$ is a contact force $-\mathbf{T}$ from the mass onto the rope.

   (c) Rope: $\mathbf{P}$ is the contact force of the pulley on the rope. The reaction force to $\mathbf{P}$ is the contact force $-\mathbf{P}$ of the rope on the pulley. How can I be sure that $\mathbf{P}$ points in the direction indicated in the figure? I can't, other than from experience in solving this sort of problem. The actual direction comes out of the algebraic calculation when the problem is solved. The force diagram is a qualitative tool, a mnemonic device to help you be sure that you have not forgotten a force when you set up the second law equations, not a devisement that gives you quantitative solutions.

$\mathbf{T}_1$ is the force due to the spring acting on the rope. The reaction force to $\mathbf{T_1}$ is a contact force $-\mathbf{T_1}$ from the rope acting on the spring.

$-m_R g\hat{\mathbf{k}}$ is the force of the earth's gravity on the rope. The reaction force to $-m_R g\hat{\mathbf{k}}$ is the force $mg\hat{\mathbf{k}}$ of the rope's gravity on the earth.

$\mathbf{T}_2$ is the contact force due to the left-hand-mass acting on the rope. Its reaction force is $-\mathbf{T}_2$, the force exerted by the rope on left-hand mass. $\mathbf{T}_2$ and $-m_R g\hat{\mathbf{k}}$ are drawn splayed apart for clarity; they may well actually be parallel.

2. (a) The force diagram shows the forces N, T, and $M_1 g$ acting on M_1. N is the normal force, a contact force, of the table acting on M_1. T is the tension force, a contact force, of the rope acting on M_1. $M_1 g$ is the gravitational force of the earth acting on M_1.

   (b) The force diagram shows the forces T and $M_2 g$ acting on M_2. T is the tension force, a contact force, of the rope acting on M_2. $M_2 g$ is the gravitational force of the earth acting on M_2.

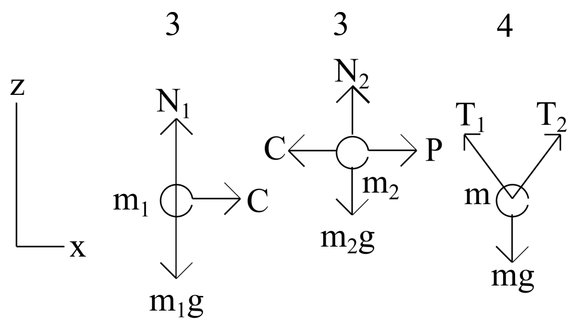

Figure 6.12: Force Diagrams for Worked Problems 3 and 4.

3. (a) The force diagram shows the forces N_1, C, and $M_1 g$ acting on M_1. N_1 is the normal force, a contact force, of the table acting on M_1. C is the contact force of M_2 pushing on M_1. $M_1 g$ is the gravitational force of the earth acting on M_1. $M_1 g$ always points vertically downward.

   (b) The force diagram shows the forces N_2, C, P, and $M_2 g$ acting on M_2. N_2 is the normal force, a contact force, of the table acting on M_2. C is the contact force of M_1 pushing on M_2. $M_2 g$ is the gravitational force of the Earth acting on M_2. P is the external force of the man pushing on M_1.

4. mg is the force of the Earth's gravity acting on the stereo speaker. T_1 and T_2 are the tension forces, contact forces where the wires touch the speaker, acting on the stereo speaker. $-mg$ is the force of the stereo's gravitational field acting on the Earth. $-T_1$ and $-T_2$ are the contact forces, where the wires touch the speaker, of the stereo speaker acting on the wires.

   Note that $-mg$, $-T_1$, and $-T_2$ are not forces applied to the speaker.

5. The Ferris Wheel

   We apply the Second Law $\mathbf{F} = m\frac{d^2\mathbf{r}}{dt^2}$, finding

$$\mathbf{N} - mg\hat{\mathbf{k}} = -\frac{mv^2}{R}\hat{\mathbf{R}}. \tag{6.32}$$

   Solving for $\mathbf{N}$, this equation becomes

$$\mathbf{N} = mg\hat{\mathbf{k}} - \frac{mv^2}{R}\hat{\mathbf{R}}. \tag{6.33}$$

At the three points A, B, and C, $\hat{\mathbf{R}}$ is $+\hat{\mathbf{k}}$, $+\hat{\mathbf{i}}$, and $= -\hat{\mathbf{k}}$, respectively. At point A, the force due to the seat is

$$\mathbf{N} = (mg - \frac{mv^2}{R})\hat{\mathbf{k}}, \tag{6.34}$$

so the force due to the seat is less than would be expected from the rider's weight. At point C, the force due to the seat is

$$\mathbf{N} = (mg + \frac{mv^2}{R})\hat{\mathbf{k}}, \tag{6.35}$$

so the force due to the seat is more than would be expected from the rider's weight. In both cases, $\mathbf{N}$ is parallel or antiparallel to the radius vector $\mathbf{R}$.

Point B is a bit more interesting. At point B,

$$\mathbf{N} = mg\hat{\mathbf{k}} - \frac{mv^2}{R}\hat{\mathbf{i}}. \tag{6.36}$$

This $\mathbf{N}$ has both horizontal and vertical components, and is not parallel or antiparallel to $\mathbf{R}$.

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